

What you'll Learn About

How to take the derivative of a function that is not solved for  $y$  (an implicitly defined function)

Find the derivative of the following function

A)  $x^2 + y^2 = 1$

$$y = (1-x^2)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-1/2} \cdot -2x$$

$$= -\frac{x}{\sqrt{1-x^2}}$$

B)  $x = \cos\theta \quad y = \sin\theta$

$$\frac{dy}{dx} = \frac{\cos\theta}{-\sin\theta}$$

$$\frac{d}{dx}(y^2) = 2y \left( \frac{dy}{dx} \right)$$

$$\frac{d}{dy}(y^2) = 2y \cdot 1$$

C)  $x^2 + y^2 = 1$

$$(x^2 + y^2) = 1$$

$$\frac{d}{dx}(x^2 + y^2) = 0$$

$$\underline{-2x + 2y \cdot \frac{dy}{dx}} = -2x$$

$$\frac{2y \frac{dy}{dx}}{2y} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y} = \frac{-x}{\sqrt{1-x^2}}$$

D)  $x^2 + y^2 = xy$

$$\frac{2x + 2y \left( \frac{dy}{dx} \right)}{-2x} = x \frac{dy}{dx} + y(1)$$

$$\underline{2y \frac{dy}{dx} = x \frac{dy}{dx} + y - 2x}$$

$$\frac{2y \frac{dy}{dx}}{-x \frac{dy}{dx}} = \frac{x \frac{dy}{dx}}{-x \frac{dy}{dx}} + \frac{y - 2x}{-x}$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

$$\frac{2y \frac{dy}{dx} - x \frac{dy}{dx}}{(2y - x)} = y - 2x$$

$$\cancel{\frac{dy}{dx}(2y - x)} = \frac{y - 2x}{2y - x}$$

$$E) \quad x^2 = \frac{x-y}{x+y}$$

$$(x+y)^2 \cdot 2x = (x+y)\left(1 - \frac{dy}{dx}\right) - (x-y)\left(1 + \frac{dy}{dx}\right) \cancel{(x+y)^2}$$

$$2x(x+y)^2 = x - x\frac{dy}{dx} + y - y\frac{dy}{dx} - \left[ x + x\frac{dy}{dx} - y - y\frac{dy}{dx} \right]$$

$$2x(x+y)^2 = \cancel{x} - \cancel{x\frac{dy}{dx}} + \cancel{y} - \cancel{y\frac{dy}{dx}} - \cancel{x} - \cancel{x\frac{dy}{dx}} + \cancel{y} + \cancel{y\frac{dy}{dx}}$$

$$\underline{2x(x+y)^2 = -2x\frac{dy}{dx} + 2y} \quad \frac{2x(x+y)^2 - 2y}{-2x} = \frac{-2x\frac{dy}{dx}}{-2x}$$

$$E) \quad x + \tan(xy) = y$$

$$-\cancel{(x+y)^2} + \frac{y}{x} = \frac{dy}{dx}$$

$$x + \tan(xy) = y^3$$

$$1 + \sec^2(xy) \cdot \left[ x\frac{dy}{dx} + y(1) \right] = 3y^2 \frac{dy}{dx}$$

$$1 + x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy) = 3y^2 \frac{dy}{dx}$$

$$- x\sec^2(xy)\frac{dy}{dx}$$

$$1 + y\sec^2(xy) = 3y^2 \frac{dy}{dx} - x\sec^2(xy) \frac{dy}{dx}$$

$$1 + y\sec^2(xy) = \frac{dy}{dx} (3y^2 - x\sec^2(xy))$$

$$\frac{1 + y\sec^2(xy)}{3y^2 - x\sec^2(xy)} = \frac{dy}{dx}$$